

Progression in Calculations: ADDITION

BAND 1

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation.

Children use pictures, stories and songs and use objects and materials to help.



They use their counting skills to find one more than, using resources to help count.

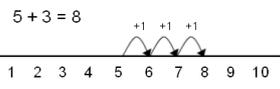
They use models and images to support learning number bonds to 10, moving on to 20 (including numicon)

Introduce missing number problems e.g. $7 = _ - 9$

They physically jump along a number line.

Written number sentences are introduced with the appropriate mathematical symbols.

Teachers start to introduce the use of the number line to support with adding 1-digit and 2-digit number to 20.



BAND 2

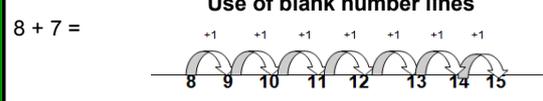
Developing a range of mental methods

Use concrete objects, including base 10, and pictorial representations to add.

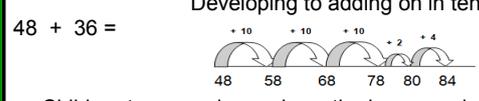
Use an apply number bonds and place value knowledge to recall facts e.g. $2 + 3 = 5$ so $20 + 30 = 50$.

Children to add in multiples of tens and ones on number squares, preparing to calculate mentally using partitioning and place value.

Use of blank number lines



Developing to adding on in tens

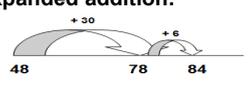


Children to recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing numbers problems. i.e. $44 + _ = 100$ ▶ $100 - 44 = _$

BAND 3

Continue to develop efficiency with a number line then introduce column method with expanded addition:

Children to cement understanding of partitioning through adding multiples of ten. $48 + 36 =$



Children to add numbers with **up to** 3 digits using formal methods.

No bridging required

HTU	
143	
+254	
7	(3 + 4)
90	(40 + 50)
300	(100 + 200)
397	

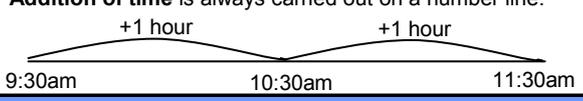
Bridging needed

TU	
57	
+35	
12	(7 + 5)
80	(50 + 30)
92	

Children to estimate answer to calculation and use inverse to check.

(Leave one clear square before the bracket.)

Addition of time is always carried out on a number line.



BAND 4

Compact Method

Children to add numbers with **up to** 4 digits using formal methods.

Adding

- First 1's
- then 10's
- then 100's
- Then 1000's

Th	H	T	U
3	2	6	7
+			
	5	8	6
3	8	5	3
1			

'Carry the hundred'

Children to continue to estimate answer to calculation and use inverse to check.

Column headings may be left off at the teacher's discretion.

BAND 5

Compact Method

Children to add whole numbers with more than 4 digits (extending to more than two numbers) using formal methods.

Th	Th	H	T	U
6	4	2	5	3
+				
	4	4	7	9
6	8	7	3	2
1				

'Carry the hundred'

Extend to decimals in the context of money and measure.

Th	H	T	U	.	t	h
£2	4	4	2	.	4	9
+						
		£7	3	.	4	2
£2	5	1	5	.	9	1
1						

'Carry the hundredths'

Children to use rounding to check answers to calculations and determine, in the context of the problem, levels of accuracy.

Column headings may be left off at the teacher's discretion.

BAND 6

Compact Method

Extend addition to more than two numbers and decimal numbers including complex decimal quantities.

Th	H	T	U	.	t	h
£4	6	8	1	.	90	
	£2	6	.	85		
+						
		£0	.	72		
£4	7	0	9	.	47	
1						

Children to solve multi-step word problems involving numbers of this difficulty.

$1.05 + 0.8 + 0.405$
Encourage children to use zero as a place holder to improve their accuracy.

U	.	t	h	t	h
1	.	0	5	0	
	0	.	8	0	0
+					
	0	.	4	0	5
2	.	2	5	5	
1					

Progression in Calculations: SUBTRACTION

BAND 1

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation.

Children use pictures, stories and songs and use objects and materials to help.

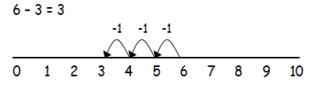
They count and point using objects, physically moving them. They respond to questions like – 'I have 3 balloons, 2 burst. How many are left?'



They use their counting skills to find one less than, using resources to help count (up to 100). They physically jump along a number line

Written number sentences are introduced with the appropriate mathematical symbols

Teachers start to introduce the use of the number line.



Develop use of the number line by counting back initially with numbers below 20 e.g. 17 - 5. Then move onto larger numbers. Use your discretion to decide when it is time to move from a marked to an empty number line.

BAND 2

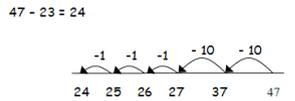
Developing the use of blank number line

Use concrete objects, including base 10, and pictorial representations to subtract.

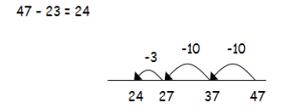
Children to subtract in multiples of tens and ones on number squares, preparing to calculate mentally using partitioning and place value. Use an apply number bonds and place value knowledge to recall facts e.g. $5 - 2 = 3$ so $50 - 20 = 30$.

First count back in tens and ones.

Note: It is vital that children can count back in tens from any number.



Then help children to become more efficient by subtracting the units in one jump (by using the known fact $7 - 3 = 4$).

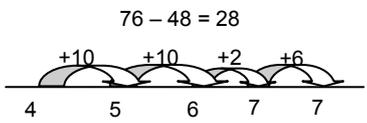


Children to recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing numbers problems. i.e. $100 - 44 = \underline{\quad}$ \blacktriangleright $44 + \underline{\quad} = 100$

BAND 3

Continue to develop efficiency with a number line then introduce column method with expanded subtraction:

The children to be introduced to complementary addition method for subtraction. The use of models, such as Dienes or Cuisenaire, is extremely important here to understand the idea of "difference"



Expanded Written Method

Children to continue to estimate answer to calculation and use inverse to check.

Children to subtract numbers with up to 3 digits using formal methods. Building understanding of place value and ability to partition numbers

No exchanging required

Exchanging needed

$89 = 80 + 9$	Step 1	$\begin{array}{r} 70 & 1 \\ - 40 & 6 \\ \hline 30 & 5 \end{array}$	<p>'one take away six' or 'take 6 from 1'</p> <p>'exchange a ten'</p>
$- 57 = 50 + 7$	Step 2	$\begin{array}{r} 60 & 11 \\ - 40 & 6 \\ \hline 20 & 5 \end{array} = 25$	

BAND 4

Developing the expanded written method to use with up to 4 digit numbers

Children to subtract numbers with up to 4 digits using formal methods.

Children to continue to confidence in expanded method including mixed numbers i.e. $4523 - 312 =$

Develop understanding of place value through exchanging with the use of physical resources if necessary; such as dienes

$$\begin{array}{r} 600 \\ - 200 \\ \hline 400 \end{array} + \begin{array}{r} 140 \\ - 80 \\ \hline 60 \end{array} + 14 + 6 = 468$$

Children to continue to estimate answer to calculation and use inverse to check.

BAND 5

Compact Method

Children to subtract whole numbers with more than 4 digits using formal methods.

$$\begin{array}{r} 3754 - 286 \\ \underline{- 286} \\ 3468 \end{array}$$

Extend to decimals in the context of money and measure.

$$\begin{array}{r} \text{£ } 3874.15 \\ \underline{- \text{£ } 7371.49} \\ \text{£ } 31371.96 \end{array}$$

Children to use rounding to check answers to calculations and determine, in the context of the problem, levels of accuracy.

BAND 6

Compact Method

Children to develop subtraction with mixed decimal up to 3dp
E.g. $4381.72 - 427.496$

$$\begin{array}{r} 3 \quad 7 \quad 6 \\ \text{A } 13 \quad \text{B } 1 \quad \text{C } 10 \\ \underline{- 427.496} \\ \text{£ } 3954.224 \end{array}$$

Children to use zero as a place holder

Children to solve multi-step word problems involving numbers of this difficulty.

Children to use the same method when zeros are within the number and need to exchange through it.

$$\begin{array}{r} 8 \quad 9 \\ 2 \quad \text{B } 1 \quad \text{C } 1 \\ \underline{- 224.21} \\ 2679.21 \end{array}$$

Progression in Calculations: MULTIPLICATION

BAND 1

Multiplication concepts introduced through addition and addition strategies.

Use of concrete apparatus for the children to physically count and see. Number rhymes

Mostly pictorial representations

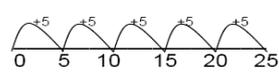
How many groups of 2 are there?



Laying the foundations for multiplying by maximizing opportunities when counting in 2's, 5's and 10's

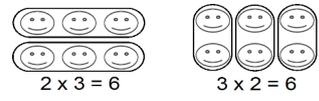
Understand multiplication as repeated addition - supported by apparatus.

$$5 \times 5 = 5 + 5 + 5 + 5 + 5$$



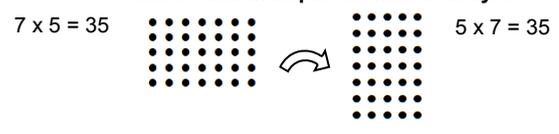
BAND 2

Understand multiplication as groups:

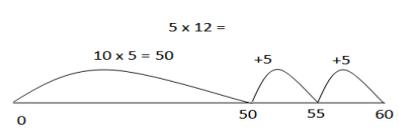


Understand commutative law:
 $4 \times 5 = 5 \times 4$

Understand multiplication as a arrays:



Develop multiplication as repeated addition through adding known multiples of a number (2's, 5's or 10's)



Recall and use x and ÷ facts for 2x, 5x and 10x tables.

BAND 3

Progression on to more formal methods
Children to multiply 2 digit by 1 digit numbers.

Introduce grid method

Use partitioning to support mental methods

Once secure, this step can left off.

$$\begin{array}{r} 24 \times 3 = \\ 20 \quad 4 \end{array}$$

$$\begin{array}{r|l} 20 & 4 \\ \hline 60 & 12 \\ \hline & 72 \end{array}$$

Work with times table facts children can do mentally in order to experience success.

$$\begin{array}{l} 4 \times 3 = 12 \\ 20 \times 3 = 60 \\ 60 + 12 = 72 \end{array}$$

Final step to support future method.

Recall and use x and ÷ facts for 3x, 4x and 8x tables.

BAND 4

Progressing to formal written methods (Expanded Column)

Children to multiply 2 digit and 3 digit by 1 digit numbers.

$$\begin{array}{r} 436 \\ \times \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 30 \\ + 150 \\ + 2000 \\ \hline 2180 \end{array}$$

First, multiply U by U

Next, U by T

Then, U by H

Finally, add answers together

Leave one square gap.

Recall x and ÷ facts up to 12 x 12

BAND 5

Develop greater understanding of expanded column method progression on to compact method.

Children to multiply up to 4 digits by a 1 or 2 digit numbers.

Expanded method

$$\begin{array}{r} 376 \\ \times \quad 43 \\ \hline 18 \quad (3 \times 6) \\ + 210 \quad (3 \times 70) \\ + 900 \quad (3 \times 300) \\ + 240 \quad (40 \times 6) \\ + 2800 \quad (40 \times 70) \\ \hline 12000 \quad (40 \times 300) \\ \hline 16168 \\ \hline \end{array}$$

Moving on to the compact method

$$\begin{array}{r} 4265 \times 6 = \\ 4265 \\ \times \quad 6 \\ \hline 25590 \\ \hline \end{array}$$

Beginning to introduce long multiplication For up to 4 by 2 digits

Children should be encouraged to estimate and check the reasonableness of their answers.

BAND 6

Compact Long Multiplication Method

Children to multiply a 4 digit by 2 digit numbers.

$$\begin{array}{r} 5628 \\ \times \quad 46 \\ \hline 33768 \\ + 225120 \\ \hline 258888 \end{array}$$

First, multiply 5628 by 6, digit at a time.

Carry over digits to be added to the next multiplied answer (ensure these are written smaller).

Next, multiply 5628 by 40, digit at a time.

Finally, add up answers.

Extend to decimals (TU.t x U and TU.t x U.t)

Progression in Calculations: DIVISION

BAND 1

Practical experience of sharing.

'One for me, one for you' is repeated subtraction of one.

The children share out toys, fruit and other materials in context where possible.

There are 12 sweets and 2 children.

They share the sweets equally, how many sweets does each child have?



Introduce Grouping

Give visual images and opportunities to physically sort objects and people into groups, e.g. There are 15 sweets and each party bag needs three sweets. How many party bags can be made?



Develop understanding of division and use jottings, including dots or tally marks, to support calculations.

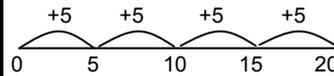
Sharing equally - 12 apples shared between 3 people, how many do they each get?



BAND 2

Beginning to use blank number lines.

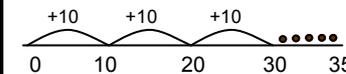
Solve grouping and sharing problems by repeated addition on an empty number line, e.g. A chew bar costs 5p. How many can I buy with 20p? $20 \div 5 =$



Cuisenaire and a number rod track make an effective model.

Begin to extend to calculations with remainders

Introduce the concept of a remainder through real situations, e.g. A chocolate bar costs 10p. How many can I buy with 35p? How much money will I have left over? $35 \div 10 = 3 \text{ r}5$

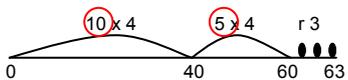


I can buy 3 chocolate bars and I will have 5p left.

Recall and use x and ÷ facts for 2x, 5x and 10x tables.

BAND 3

Consolidate calculations with remainders and developing to jump in 'chunks' of the divisor, $63 \div 4 = 15 \text{ r}3$



$$\begin{aligned} 10 \times 4 &= 40 \\ 5 \times 4 &= 20 \\ 2 \times 4 &= 8 \end{aligned}$$

Children should make a note of 10x, 5x and 2x the divisor to help them choose an appropriate 'chunk'.

As a next step, children should also record remainders as a fraction.

$$63 \div 4 = 15 \frac{3}{4}$$

Children to be able to divide numbers with 2 digits by 1 digit.

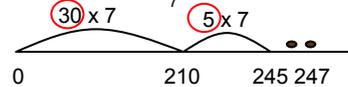
Recall and use x and ÷ facts for 3x, 4x and 8x tables.

BAND 4

Extend use of number line to larger numbers where children jump in chunks of multiples of 10.

Children to be able to divide numbers with 3 digits by 1 digit

$$247 \div 7 = 35 \text{ r}2 = 35 \frac{2}{7}$$



Once secure, link and extend to vertical 'chunking' method

$$\begin{array}{r} 50 \times 7 \\ 5 \times 7 \\ 3 \times 7 \\ \hline 412 \div 7 = 58 \text{ r}6 \\ = 58 \frac{6}{7} \end{array}$$

$$\begin{array}{r} 350 \\ + 35 \\ + 85 \\ + 21 \\ \hline 406 \\ + 6 \\ \hline 412 \end{array}$$

remainder

Children need a secure knowledge of 'tables' facts and be able to derive associated facts.
 $5 \times 7 = 35$ so
 $50 \times 7 = 350$

Keep making a note of key multiples to support.

Recall x and ÷ facts up to 12 x 12

BAND 5

Compact Written Method.

It is important that children have a clear conceptual understanding of division before moving onto the 'bus-stop' method.

Children to be able to divide numbers up to 4 digits by 1 digit

$$\begin{array}{r} 7 \text{ r}3 \\ 4 \overline{) 291} \end{array}$$

This method is easily extended into decimal remainders.

$$\begin{array}{r} 7 \text{ r}2.75 \\ 4 \overline{) 291.300} \end{array}$$

BAND 6

Compact Written Method - Long and Short division

Children to be able to divide numbers with 4 digits by 2 digit

Long Division (4 by 2)

Divide one number at a time by 23, starting with Th

$$\begin{array}{r} 114 \text{ r}20 \\ 23 \overline{) 2642} \\ \underline{-23} \\ 34 \\ \underline{-23} \\ 112 \\ \underline{-92} \\ 20 \end{array}$$

$4 \times 23 = 92$

Short Division (4 by 2)

Divide one number at a time by 23, starting with Th

$$114 \text{ r}20 \text{ or } 114 \frac{20}{23}$$

$$23 \overline{) 2642}$$